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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Hessert Center for Aerospace Research, The University of Notre Dame, Notre Dame, IN 46556-5684	8. PERFORMING ORGANIZATION REPORT NUMBER 1
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13. ABSTRACT (Maximum 200 words)

This research effort has as its objectives: the experimental characterization and low order modeling of the coherent structure dynamics of high Reynolds number rectangular jets of low and moderate aspect ratios via a joint application of the proper orthogonal decomposition (POD) and wavelet analysis techniques. This includes the extraction and characterization of the u, v, and w-component POD eigenfunctions and associated eigenvalues. The rectangular jet dynamics (embodied in the POD temporal coefficients) is obtained by projection of the POD eigenfunctions onto instantaneous realizations of the flow field. This projection is performed via wavelet analysis. Similarly, Galerkin projection of a truncated subset of the empirical eigenmodes onto the Navier-Stokes equations is used to develop a low-order dynamical systems model of the rectangular jet flow field. The validity of assumptions associated with the treatment of the truncated modes and their effect on the coherent structure is established by comparing the dynamic behavior resulting from the model ODE's with those from projection of POD modes onto the actual flow. The dynamical system model of the rectangular jet will be utilized for the establishment of rectangular jet flow control strategies. The flow control algorithm will be demonstrated via both the model and laboratory jet flows.

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Dynamic Modeling and Flow Control Strategies for Rectangular Jets:  
Application of POD and Wavelet Analysis

AFOSR Grant F49620-00-1-0013

Final Project Report

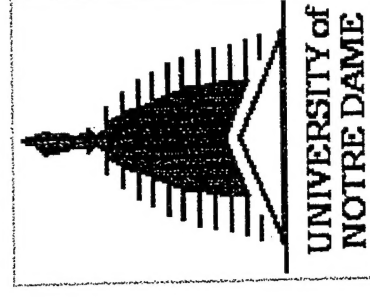
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**Start Date: November 15, 1999**

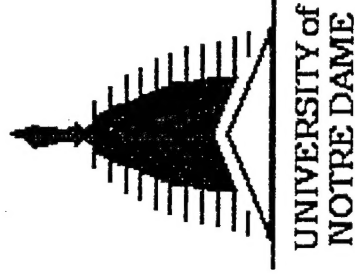
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*AFOSR Contractors' Meeting on Turbulence, 8/16/2000*

# Organization

- Motivation and Objectives
- Approach
- Experimental Results
- Modeling Results
- Plans for the Upcoming Year
- Questions / Discussion



# Motivation

- It is widely recognized that coherent vortical structures in turbulent flows are important in processes such as *scalar and momentum transport, mixing and noise generation*.
- The routine use of the coherent structure concept in turbulence modeling and flow control strategies has not yet been realized.
- Several recent successes at addressing coherent structure topology and dynamics have been achieved by the joint application of the proper orthogonal decomposition (POD) and dynamical systems theory.

## •Example “success stories”:

Near-wall region of turbulent boundary layer,

Aubry, N., Holmes, P., Lumley, J. L. and Stone, E., 1988

Asymptotic region of plane mixing layer

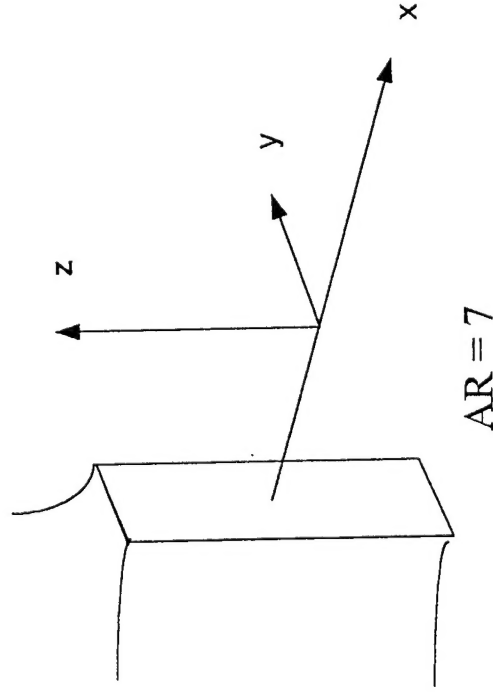
Ukeiley L. and Glauser, M. N. , 1995

## Objectives

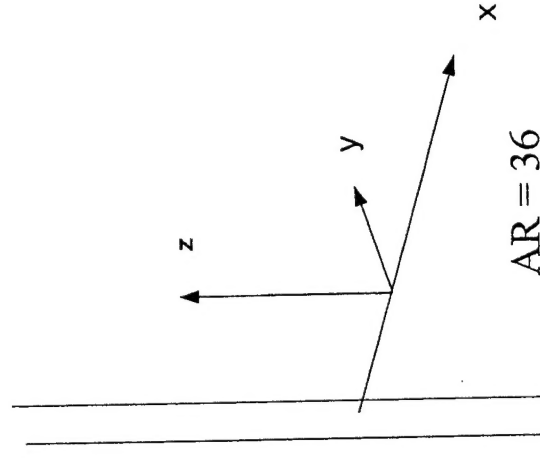
- The experimental characterization and low-order modeling of the coherent structure dynamics of rectangular jets of low and moderate aspect ratios via a joint application of POD and wavelet analysis.  
(Rectangular jets are of interest from the standpoint of both thrust vectoring and low observables.)
- The resulting dynamical system model of the rectangular jet will be utilized for the establishment of rectangular jet flow control strategies.

## Approach

- Perform initial year-one proof-of-concept experiments and modeling in the *far field of a planar jet*. These are considered prerequisite to the treatment of the more complex rectangular jet.



Two inhomogeneous spatial coordinates ( $y, z$ )



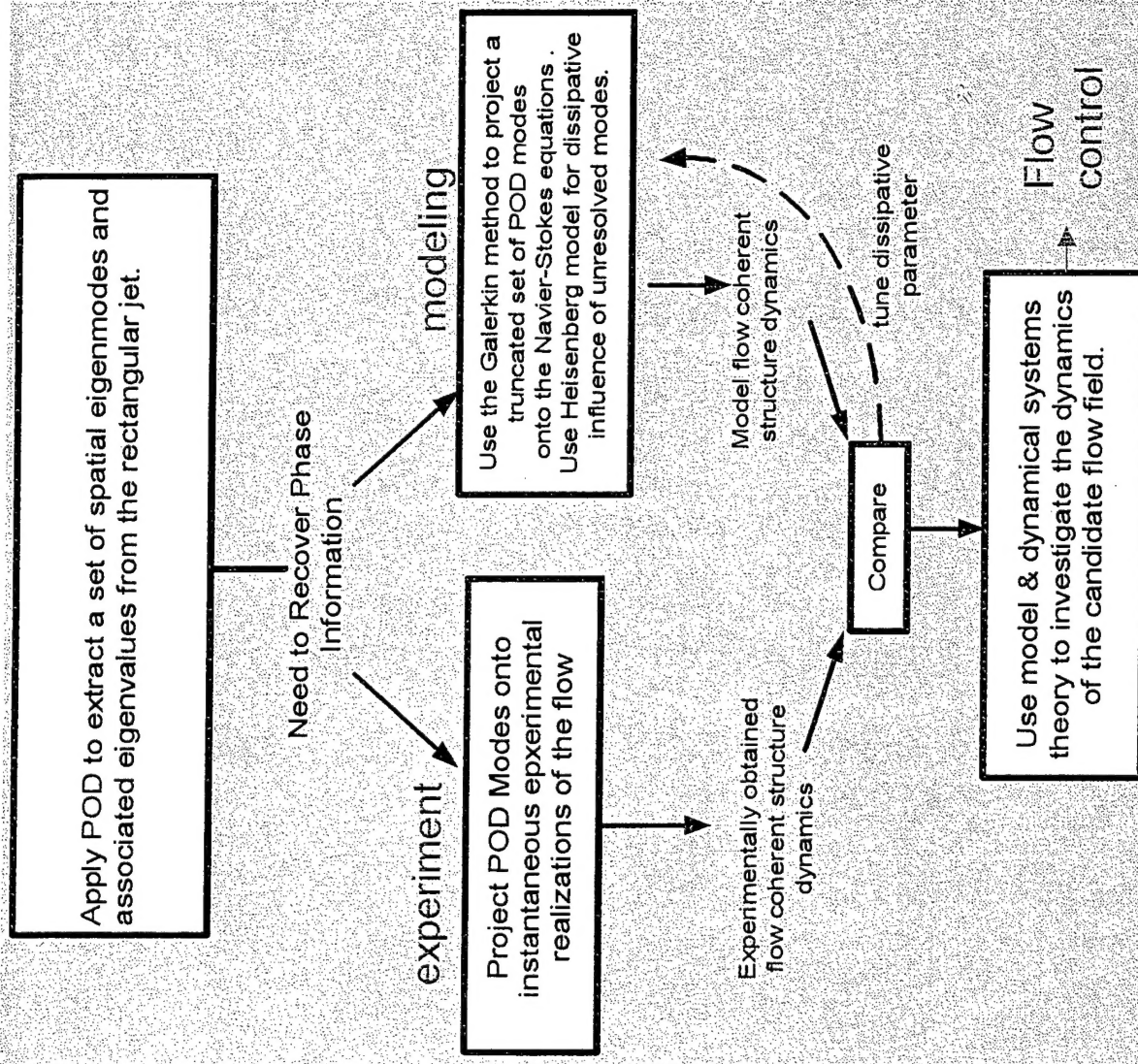
Single inhomogeneous spatial coordinate,  $y$

## Approach

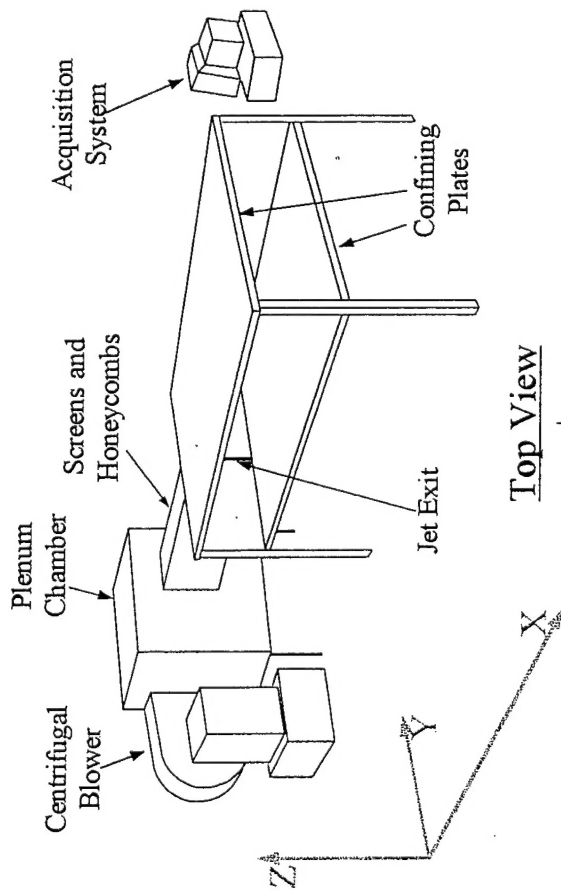
- The measurements were performed in the fully developed part of the flow ( $50 \leq x / D \leq 90$ ) where visualization by injection of a tracer reveals no evidence of underlying coherent structure.
- *This made an ideal test case to investigate the potential of the method for extraction and modeling of flow structure in a complex environment.*
- Approach to be taken for the investigation of the rectangular jet is fundamentally similar to that which will be described for the proof-of-concept planar jet study.



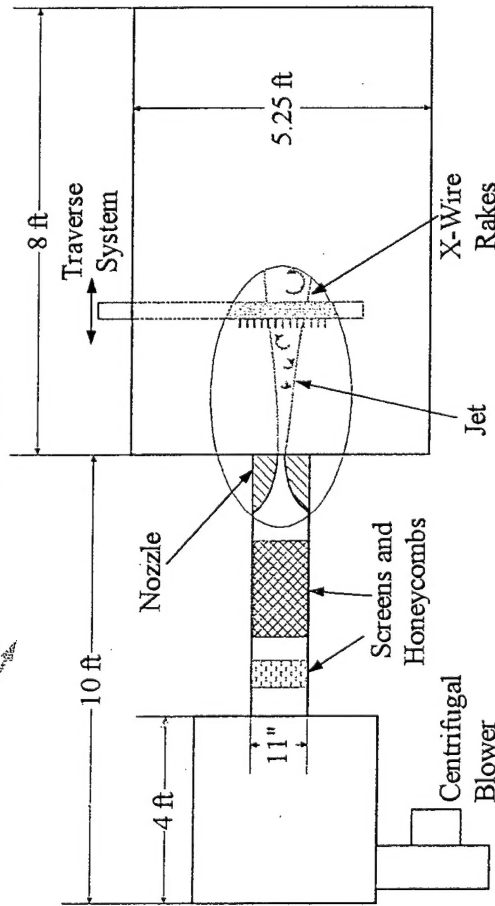
## Approach:



# Planar Jet Flow facility

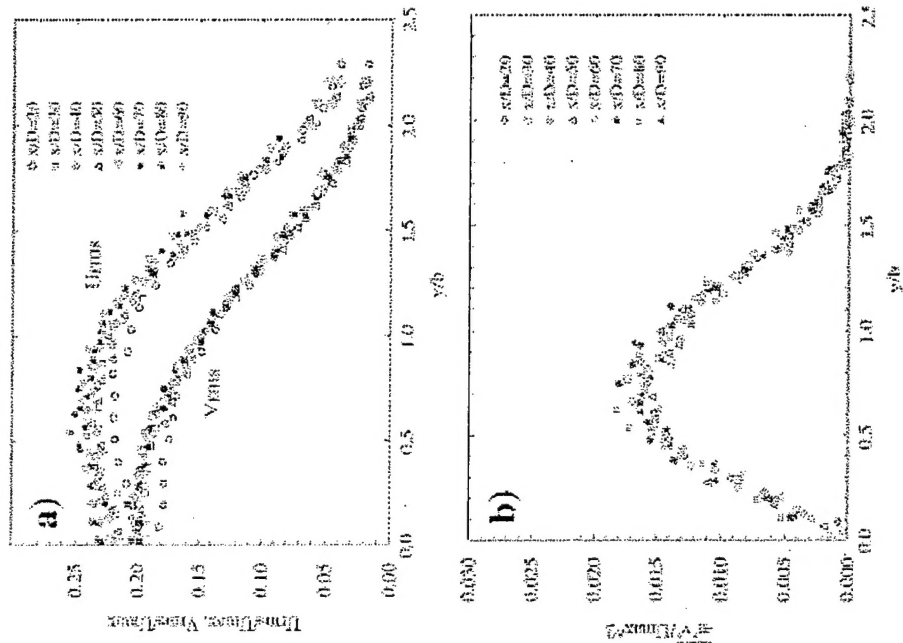


Top View

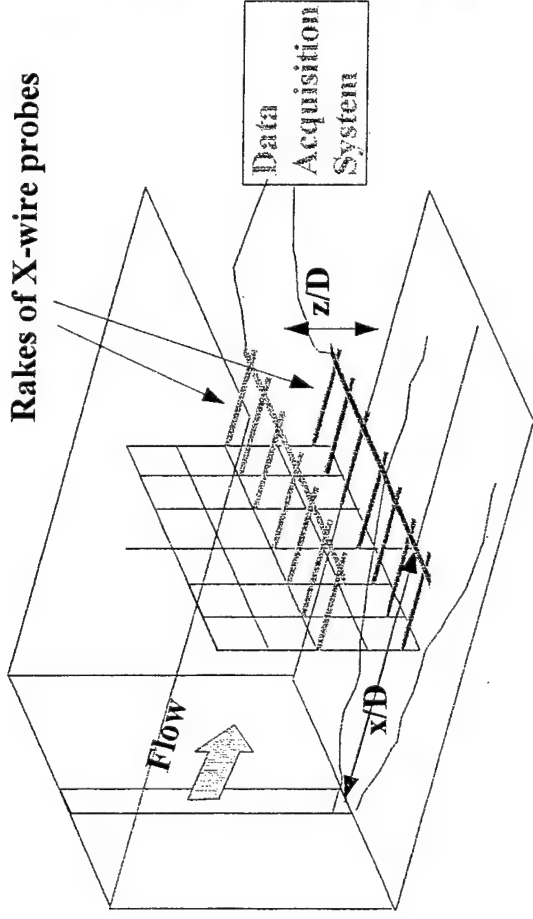
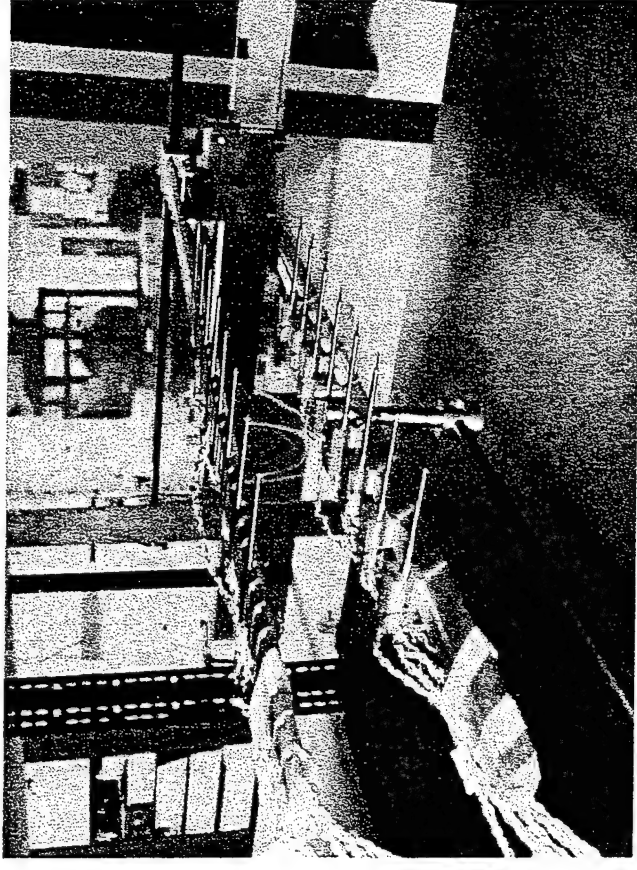


Flow exhibits self-similarity  
in second-order moments  
after  $x/D=50$

Nozzle width  $D=0.5''$   
Nozzle exit aspect ratio 36:1  
Laminar nascent free shear layers  
Initial velocity = 35 m/sec  
 $Re=28,000$

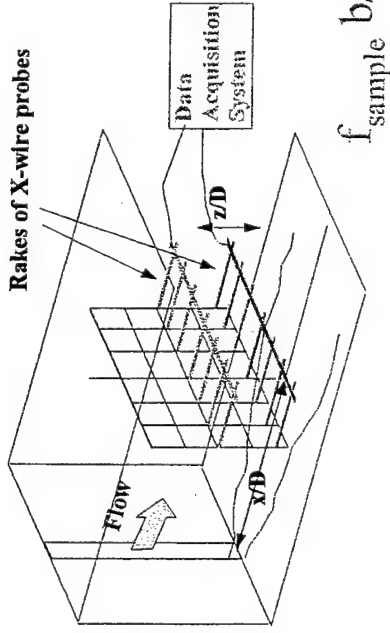


## Twin X-wire Rake Set-up



- 16 x-wire probes (32 sensors) mounted on two movable rakes.
- Spacing between probes - 2" (0.66b)
- Spacing between rakes - 15 positions in 0.75" (0.25b) increments.
- Blockage effect is demonstrated to be negligible.

# Eigenmode Extraction



Instantaneous velocity components

$$u_\alpha(y, z, t)$$

FT in time (500 blocks of 1024 points each)

$$\hat{u}_\alpha(y, z, f) = \int u_\alpha(y, z, t) e^{-2\pi i f t} dt$$

Spectral Correlation Matrix  $S_{\alpha\beta}(y, y', \Delta z, f) = \langle \hat{u}_\alpha^*(y, z, f) \hat{u}_\beta(y', z + \Delta z, f) \rangle$

FT in z-direction (16 points)

$$\Phi_{\alpha\beta}(y, y'; f, k_z) = \int S_{\alpha\beta}(y, y', \Delta z, f) e^{-2\pi i k_z \Delta z} d(\Delta z)$$

$$\int \Phi_{\alpha\beta}(y, y'; f, k_z) \varphi_\beta^{(n)}(y'; f, k_z) dy' = \lambda^{(n)}(f, k_z) \varphi_\alpha^{(n)}(y; f, k_z)$$

POD equation to find eigenvalues (energy) and eigenfunctions (spatial distribution) of the modes

\* Similar to the implementation of Ukeiley & Glauser (1995)

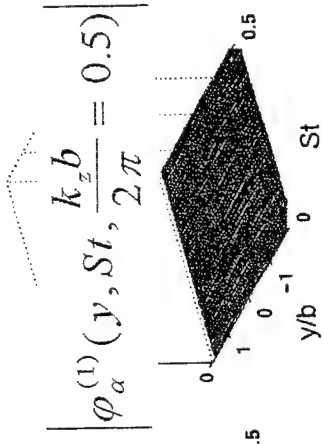
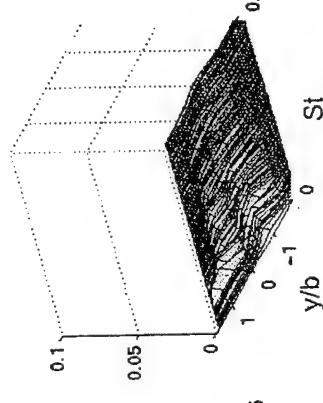
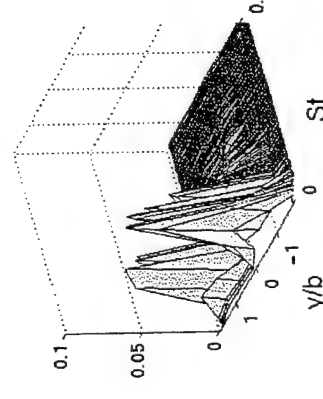
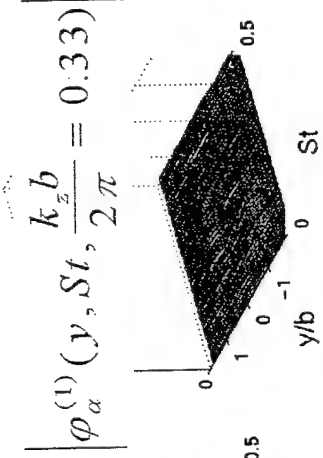
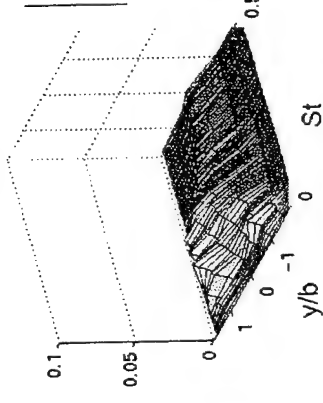
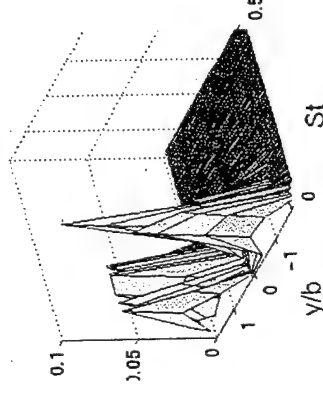
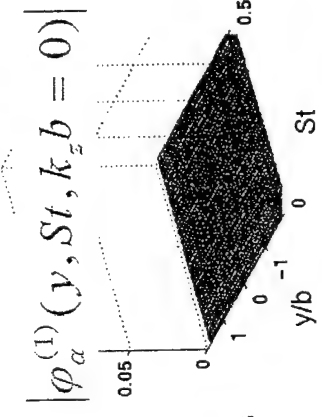
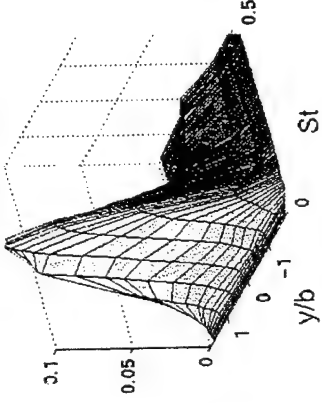
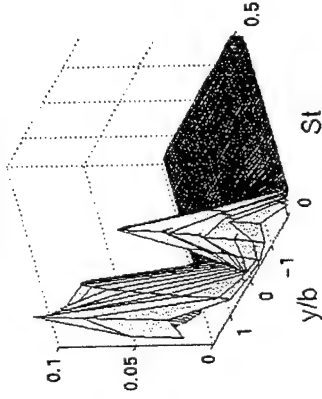
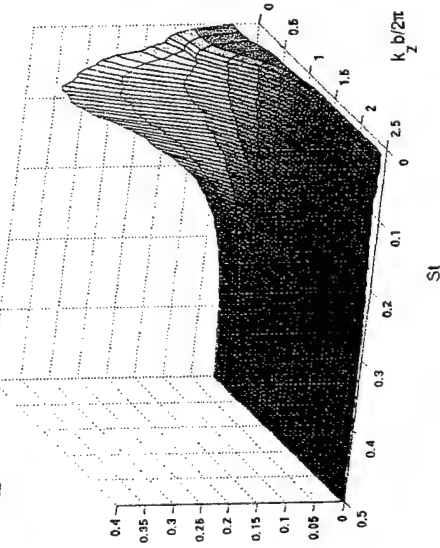
# Example: First POD Mode ( $x/D=70$ )

U-mode

V-mode

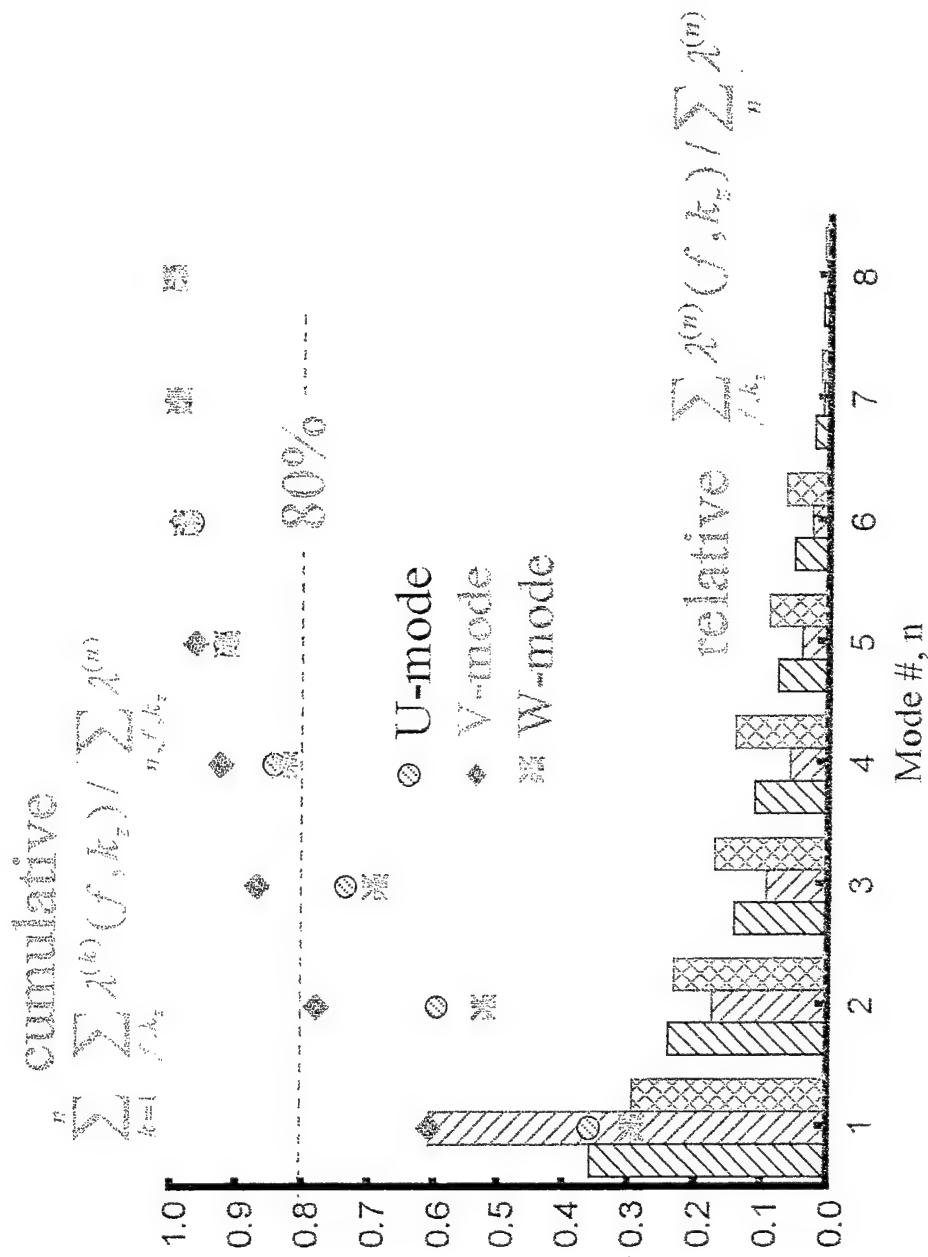
W-mode

Eigenvalue:  $\lambda^{(1)}(St, k_z b)$



• Details presented in Gordeyev and Thomas, *JFM*, 414, pp. 145-194, 2000

# Energy Convergence



- Series converges rapidly; for example:
- 4 U-modes, 2 V-modes and 4 W-modes to capture over 80% of energy of their respective component.

# Velocity Field Reconstruction via Wavelets

$$\int \Phi_{\alpha\beta}(y, y'; f, k_z) \varphi_{\beta}^{(n)}(y'; f, k_z) dy' = \lambda^{(n)}(f, k_z) \varphi_{\alpha}^{(n)}(y; f, k_z)$$

• Eigenmodes are known up to an arbitrary phase  $\exp(i\sigma(f, k_z))$

Need to have instantaneous flow field realizations

$$\tilde{u}_{\alpha}(y; k_z, \kappa(f), \tau) = \sum_n c^{(n)}(\kappa(f), \tau; k_z) \varphi_{\alpha}^{(n)}(y; f, k_z) \quad \text{- Forward Wavelet}$$

unknown phases

From orthogonality of the modes

$$c^{(n)}(k_z, \kappa(f), \tau) = \int \tilde{u}_{\alpha}(y; k_z, \kappa(f), \tau) \varphi_{\alpha}^{(n)*}(y; f, k_z) dy$$

Reconstruction of the individual mode

$$\tilde{u}_{\alpha}^{(n)}(y; k_z, \kappa(f), \tau) = c^{(n)}(\kappa(f), \tau, k_z) \varphi_{\alpha}^{(n)}(y; f, k_z)$$

↗ Inverse Wavelet ↘

Structures in physical space  $u_{\alpha}^{(n)}(y, k_z, t)$

$$u_{\alpha}(y, k_z, t) = \sum_n u_{\alpha}^{(n)}(y, k_z, t)$$

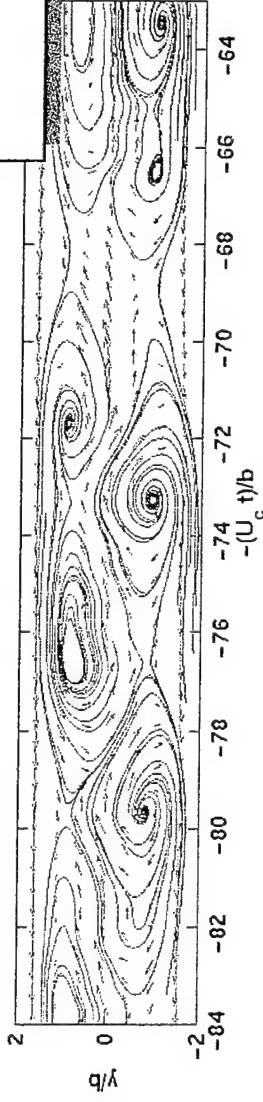
Need to perform FT in z-direction  
Measurements with 3 rakes minimum  
With 2 rakes -  $k_z=0$  only



# Velocity Reconstruction (planar mode)

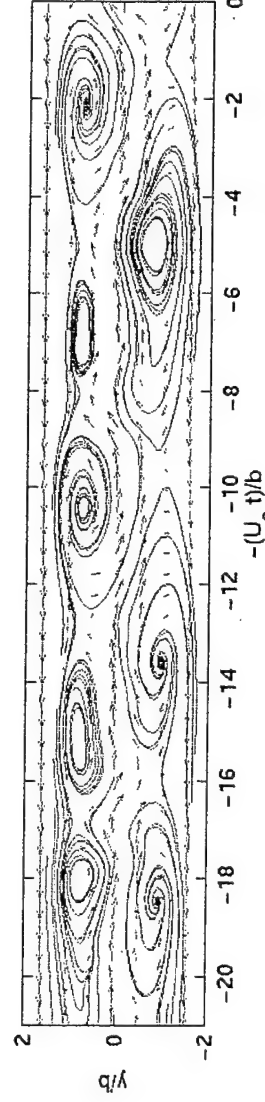
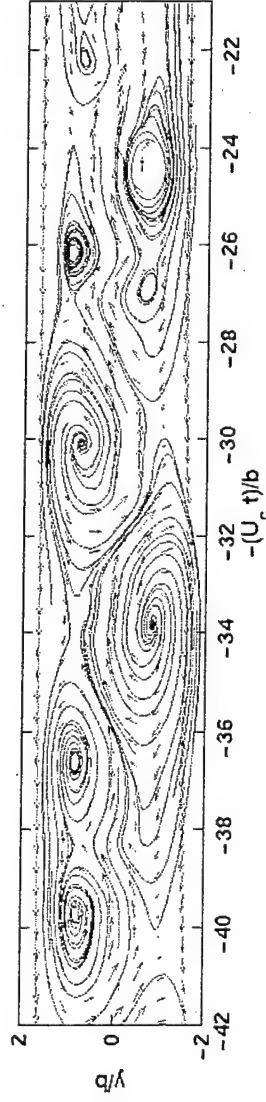
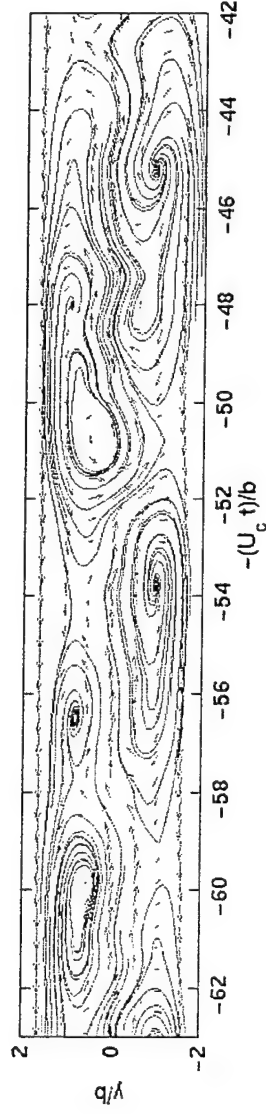
$x/D=70$

$$\bar{U}_1(y) + \sum_n u_\alpha^{(n)}(y, k_x b = 0, -U_c t)$$



First 3 planar POD modes

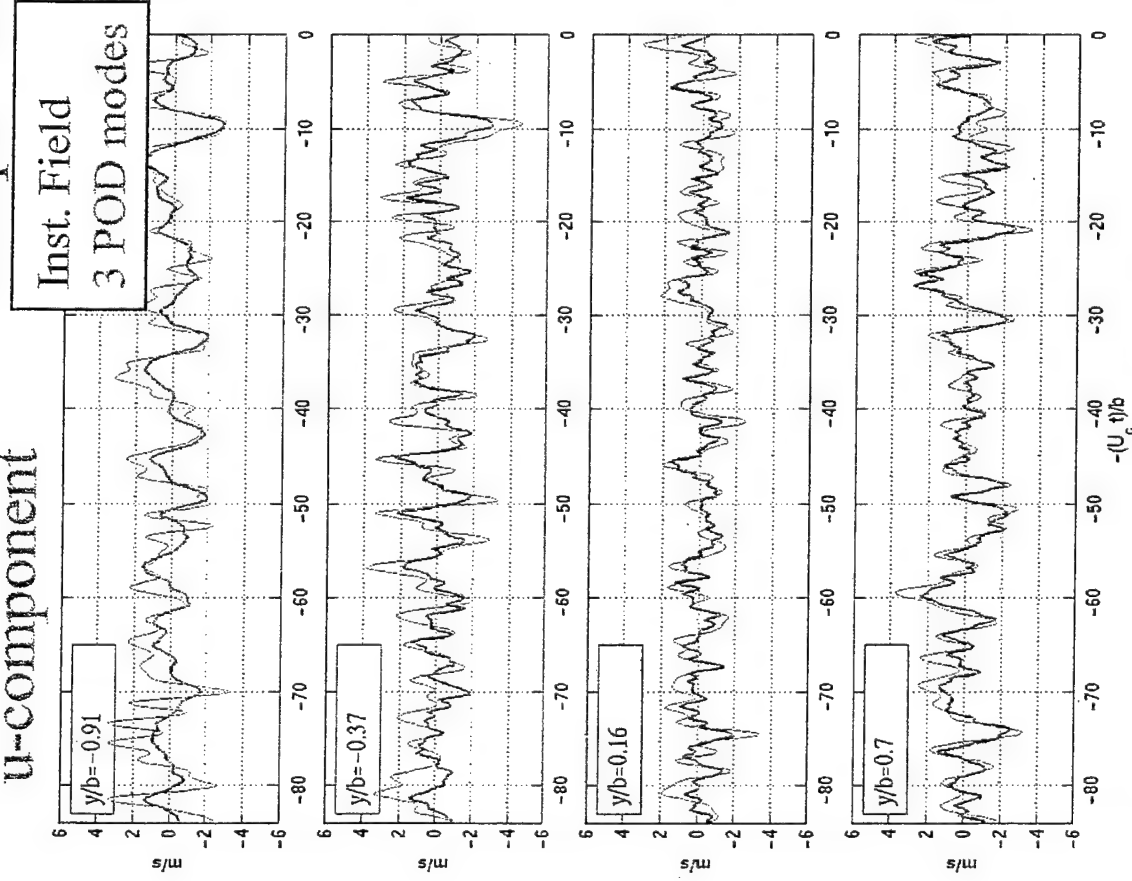
- Strong evidence of spanwise vortical structures arranged asymmetrically with respect to the jet axis.
- Karman street like organization.
- Weakly intermittent appearance.



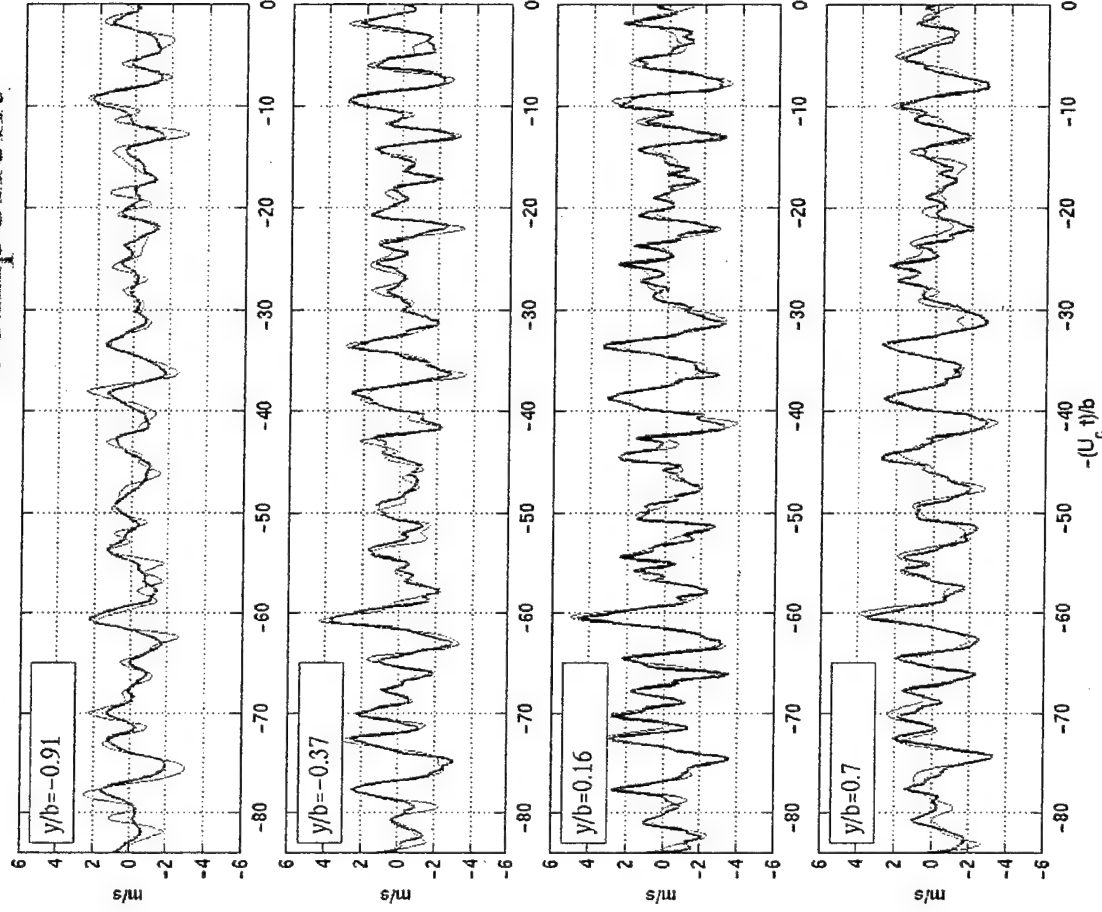


# Sample Velocity Traces

u-component

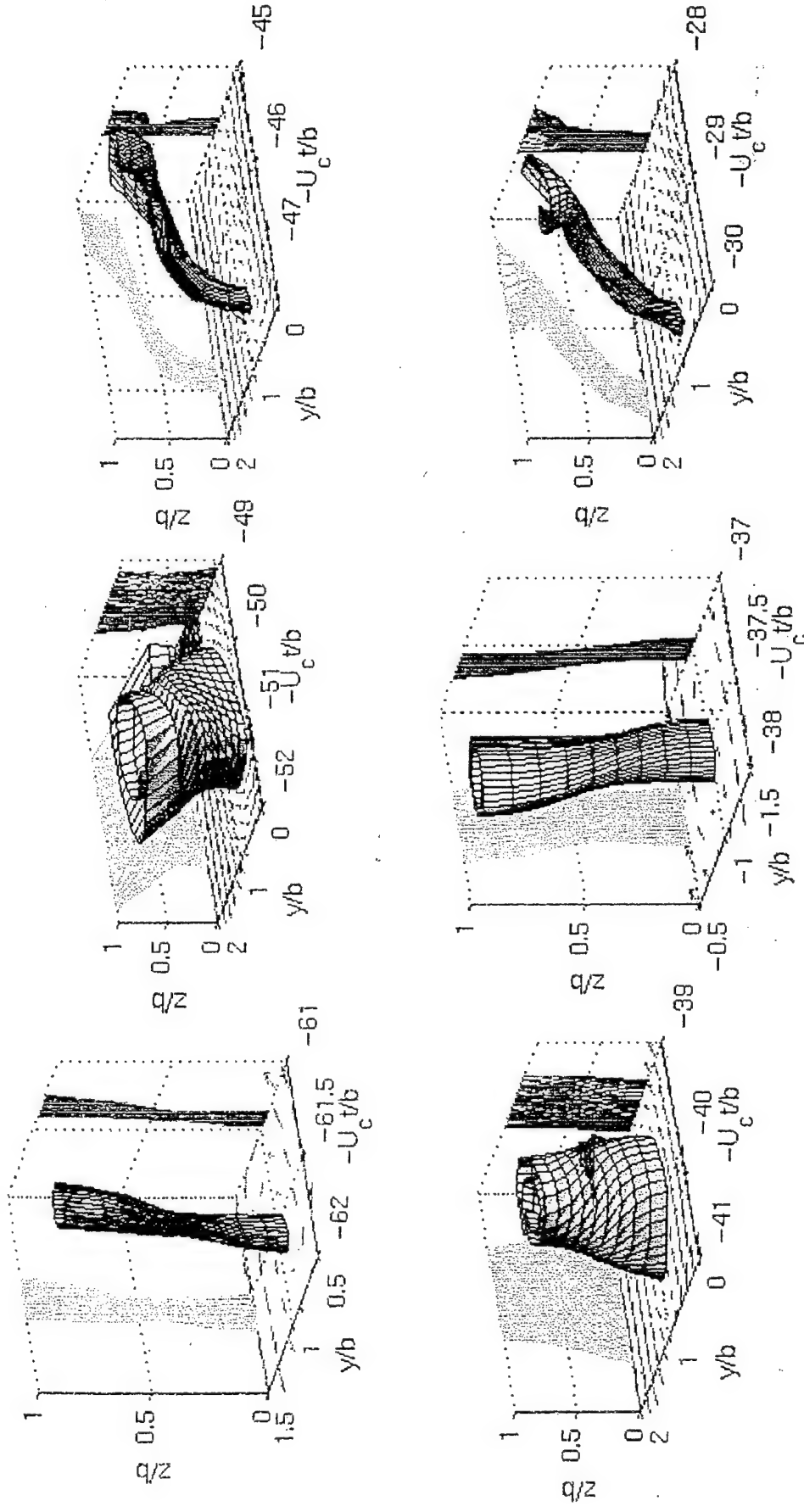


v-component



- First 3 POD modes capture velocity signals quite well.

# Reconstruction of non-planar structure



- Effect of non-planar modes is to tilt and bend the spanwise vortices in the crosstream and streamwise direction
- Non-planar modes appear to present a small disturbance to a dominant planar structure

# Galerkin Projection and Truncation

Governing Equations:  $L(u) = \frac{\partial u}{\partial t} + L(u) + Q(u, u) = 0$

Truncation:  $u^N(\vec{x}, t) = \sum_{n=1}^N a_n(t) \varphi_n(\vec{x}) \leftarrow \text{from experiment}$

Require,  $(L(u^N) \cdot \varphi_n) = 0$

Finally,  $\frac{da_n}{dt} = Ba_n + Q(a_n, a_n) + C(a_n, a_n, a_n)$   
 production interaction      interaction  
 dissipation between      between modes and  
 convection modes      mean flow

• Need to take into account energy transfer between resolved modes and unresolved turbulence (simple eddy viscosity model)

dissipation rate  $\sim \nu + \alpha \nu_T$

Aubry, Holmes, Lumley, Stone, 1988 - boundary layer

Zheng, Glauser, 1991 - round jet

Ukeiley, Glauser, 1995 - planar shear layer

## Modeling Approach

$\varphi_{\alpha}^{(n)} = \varphi_{\alpha}^{(n)}(y, k_x, k_z)$  - obtained from the experiment

### Key Assumptions:

- Homogeneous flow in x-direction,  $\bar{U}_1 = \bar{U}_1(y)$

$$t \rightarrow x = -U_c t, \quad \varphi(y, f, k_x) \rightarrow \varphi(y, -k_x, k_z)$$

- $U_c = 0.7 U_M = \text{const}$

- $U_2 = U_3 = 0$

- $n=1$  - first POD mode only

- no feedback between mean flow and resolved modes

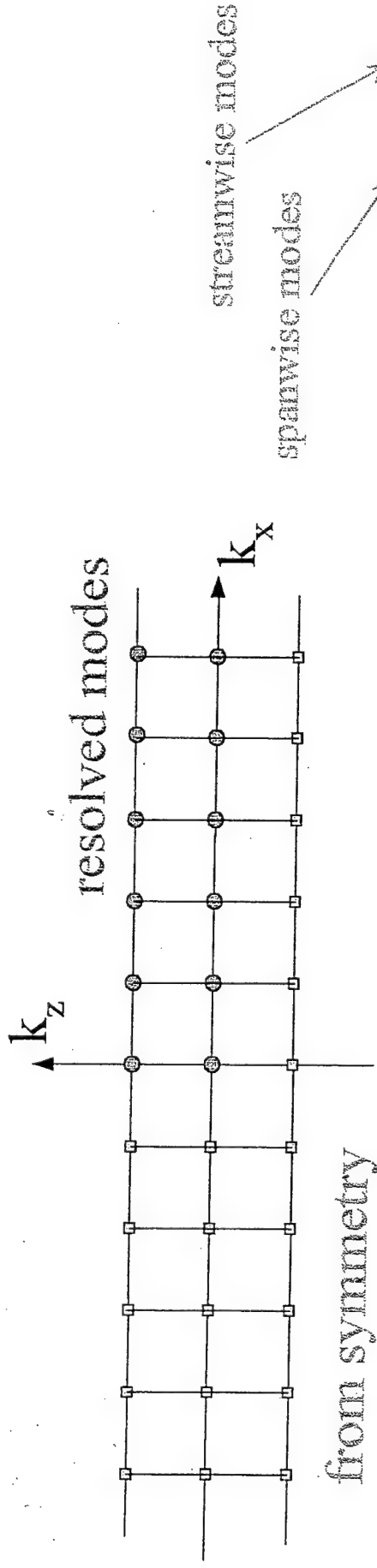
# Symmetry Conditions

$$\begin{aligned}\varphi_{u,v}^{(n)}(y, k_x, k_z) &= \varphi_{u,v}^{(n)*}(y, -k_x, k_z), \\ \varphi_w^{(n)}(y, k_x, k_z) &= -\varphi_w^{(n)*}(y, -k_x, k_z) \\ a^{(n)}(t, y, k_x, k_z) &= a^{(n)*}(t, y, -k_x, k_z)\end{aligned}$$

- translation in x-direction

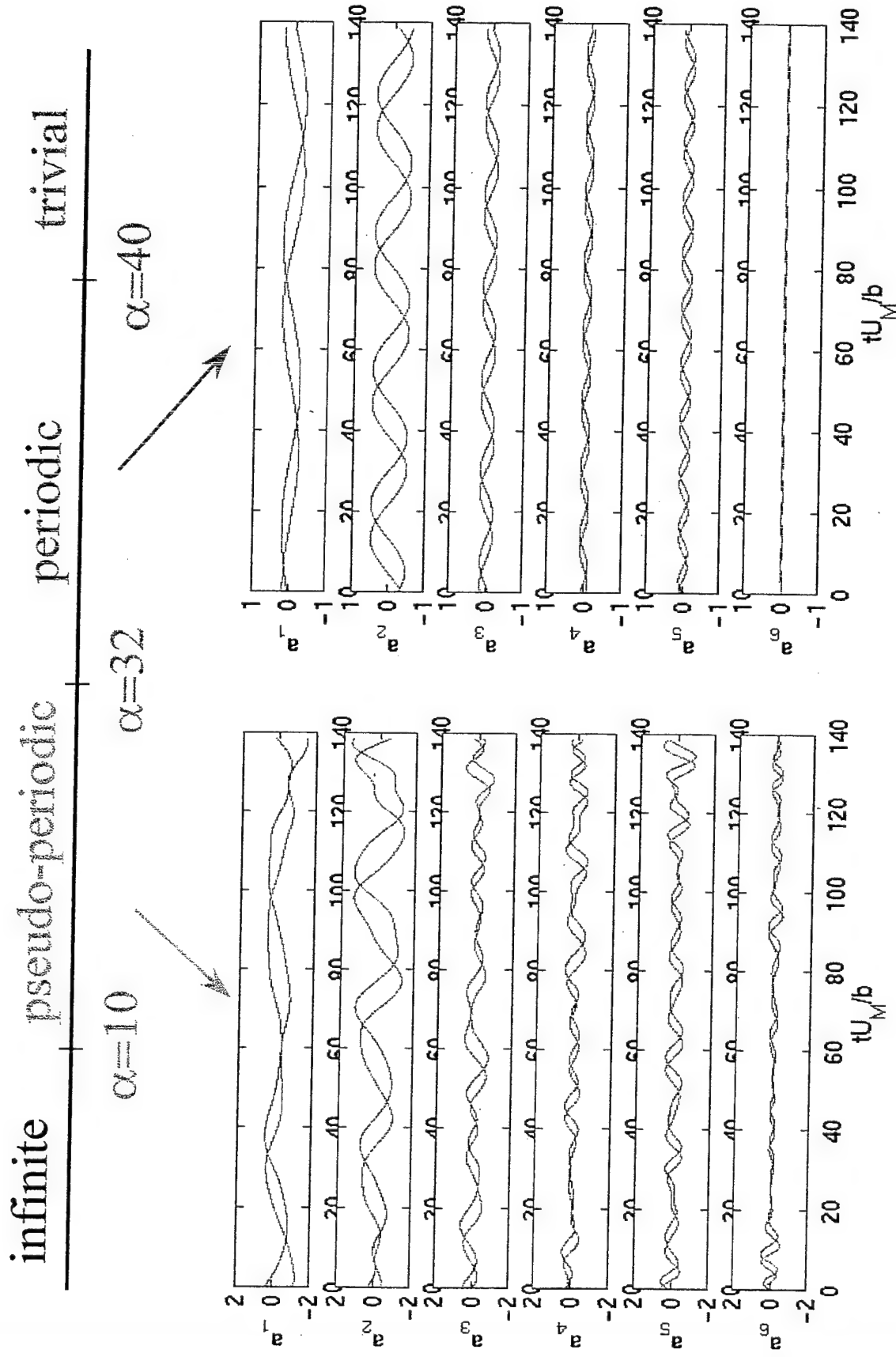
$$\begin{aligned}\varphi_{u,v}^{(n)}(y, k_x, k_z) &= \varphi_{u,v}^{(n)}(y, k_x, -k_z), \\ \varphi_w^{(n)}(y, k_x, k_z) &= -\varphi_w^{(n)}(y, k_x, -k_z) \\ a^{(n)}(t, y, k_x, k_z) &= a^{(n)}(t, y, k_x, -k_z)\end{aligned}$$

- reflection in z-direction



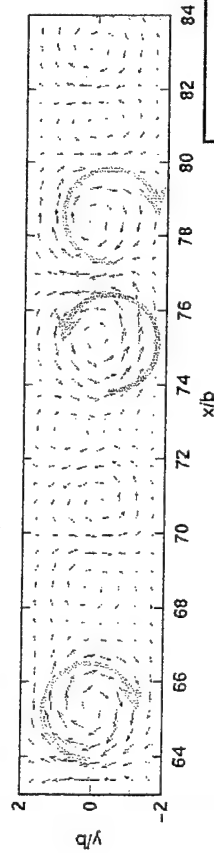
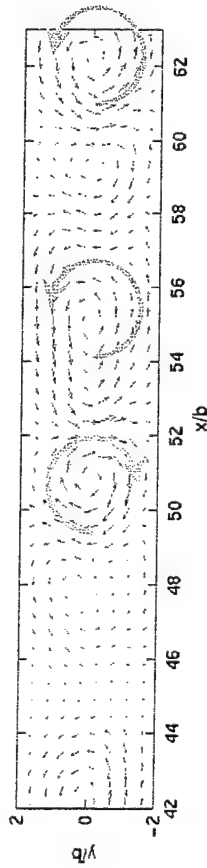
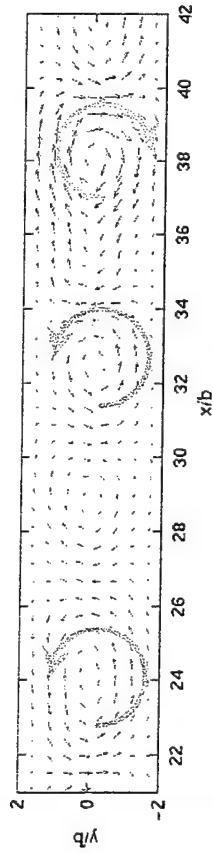
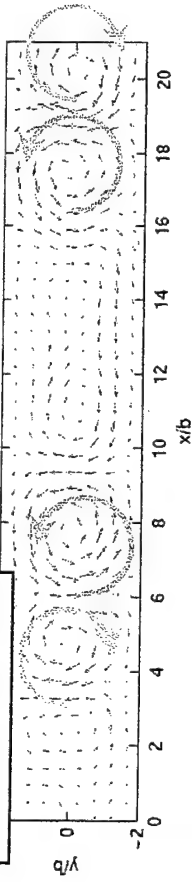
# of equations=2 x 1 x 50

# Model behavior versus $\alpha$



# Reconstructed flow (model)

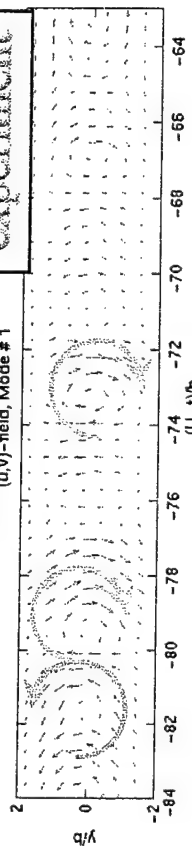
model,  $\alpha=15$



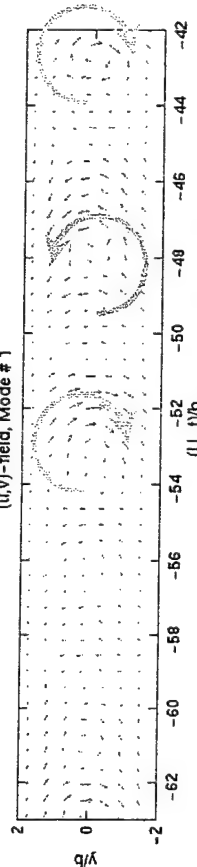
first POD mode

experiment

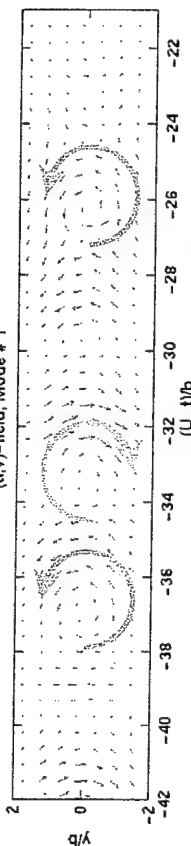
(u,v)-field, Mode #1



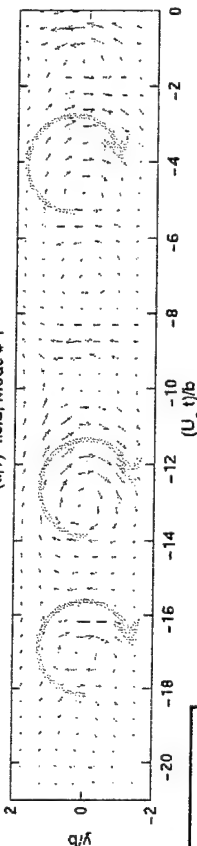
(u,v)-field, Mode #1



(u,v)-field, Mode #1



(u,v)-field, Mode #1



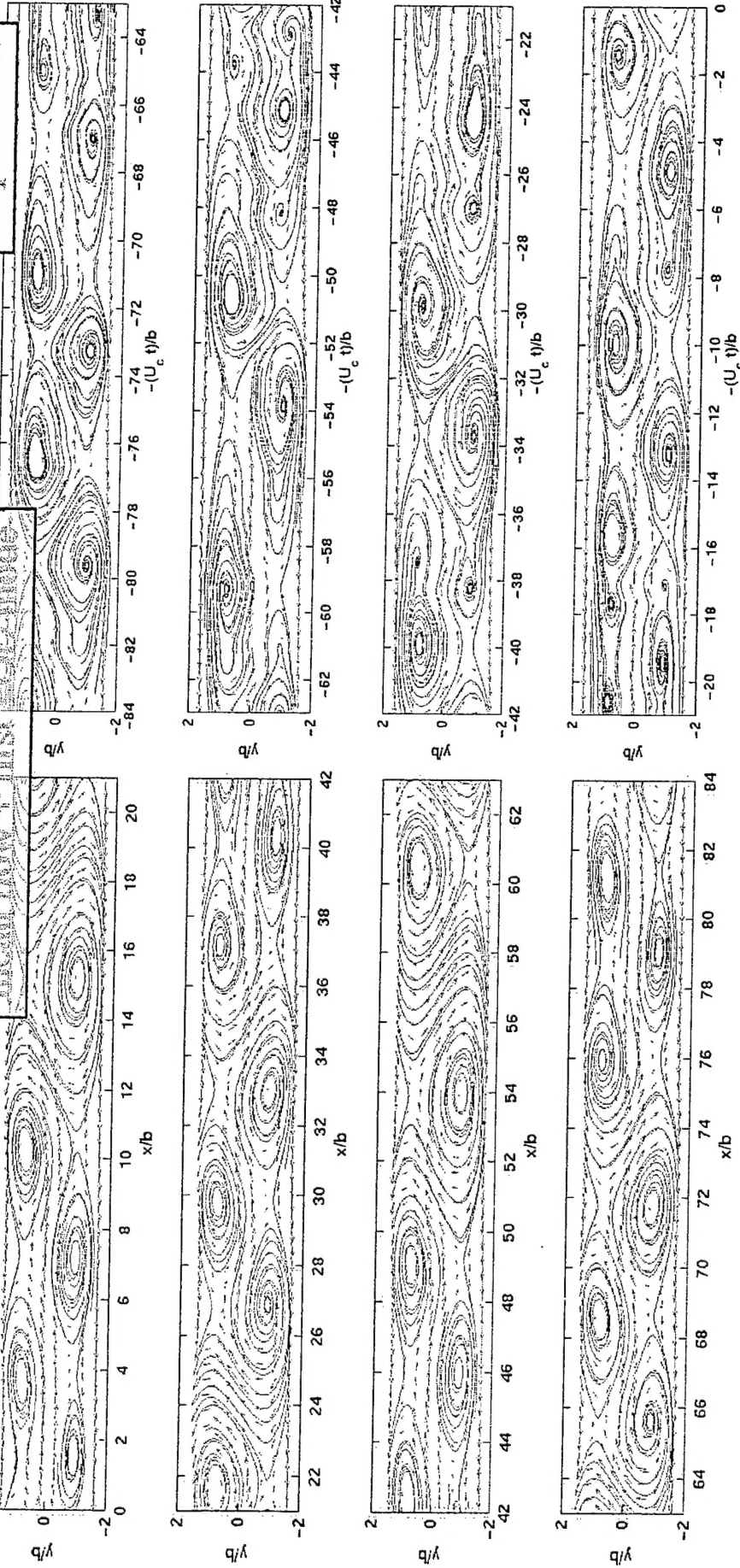
- Pseudo-periodic regime agrees very well with experimentally observed topology of large-scale coherent structures in the jet.
- Model correctly predicts lateral sweeping motions, as well as the typical size, spacing and locations of vortical structures

# Reconstructed flow (model)

Model

mean flow + first POD mode

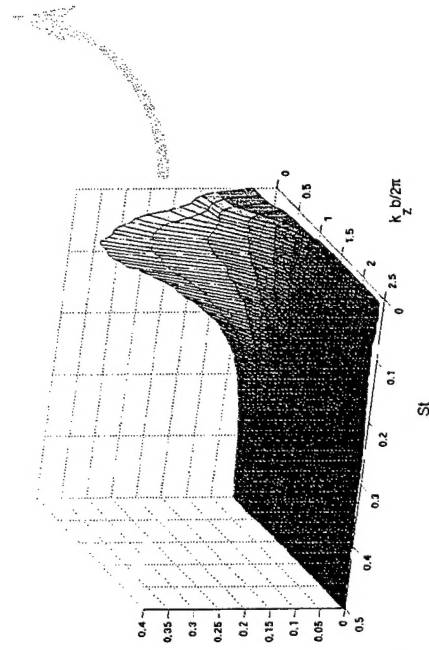
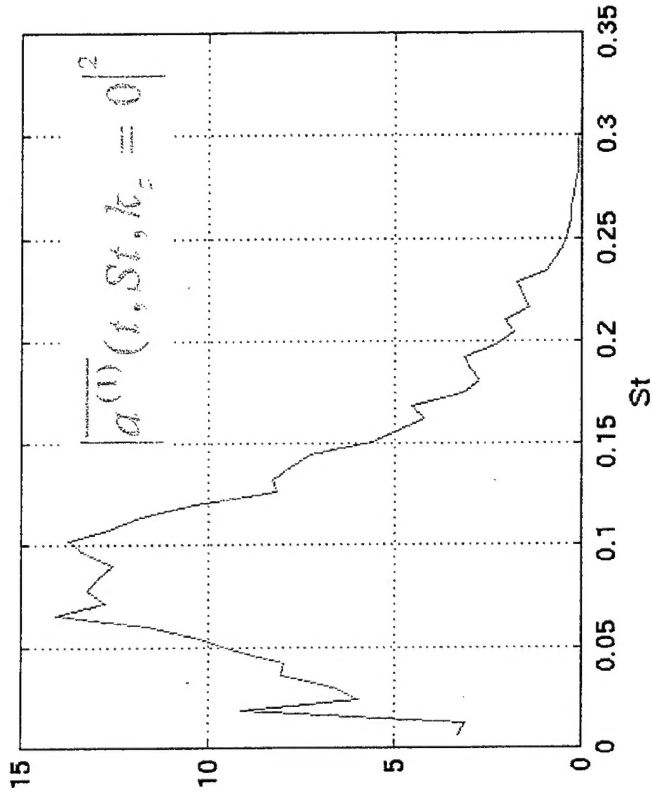
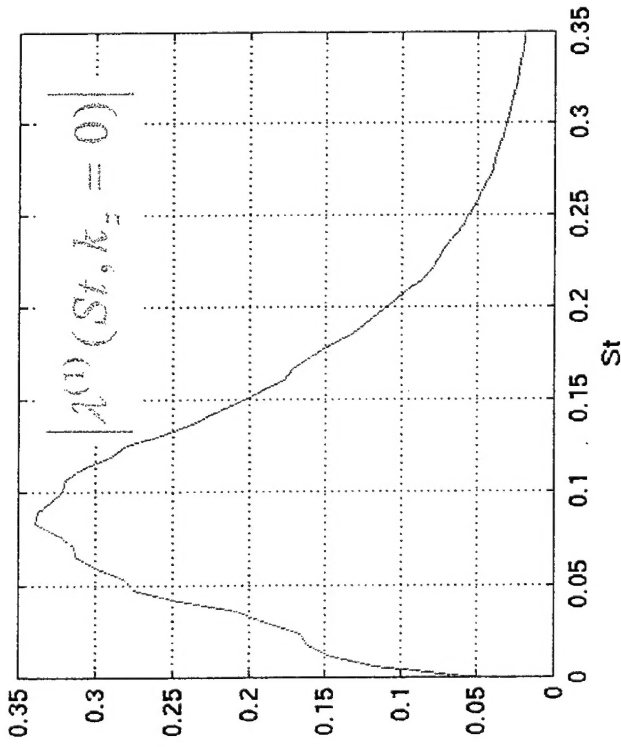
Experiment



- Pseudo-periodic regime agrees very well with experimentally observed topology of large-scale coherent structures in the jet
- Model correctly predicts lateral sweeping motions, as well as the typical size, spacing and locations of vortical structures
- Agreement between model and experiment - weak interaction between mean flow and structures



# Spectrum Recovery



- Model correctly recovers the shape of the spectrum

*AFOSR Contractors' Meeting on Turbulence, 8/16/2000*

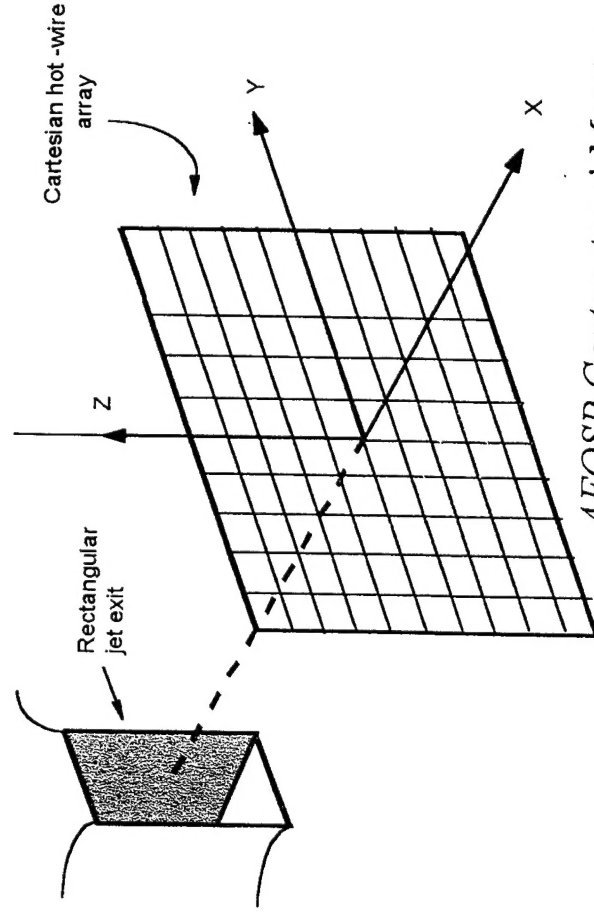
## Project Status

- Proof-of-concept experimental implementation of POD and low-order modeling of the self-similar planar turbulent jet has nearly been completed.
- Results show great promise for incorporating coherent structure dynamics in models of complex turbulent flows (like the rectangular jet of interest here).
- Presently working on the incorporation of nonplanar modes (first results obtained last week).
- Similar methodology to be applied to the  $AR=7$  rectangular jet next.

*AFOSR Contractors' Meeting on Turbulence, 8/16/2000*

## Project Status

- Rectangular jet nozzle fabricated.
- Initial hot-wire surveys at  $Re = 50,000$  now underway.
- Measurements to obtain the POD eigenfunctions  $\varphi_{\alpha}^{(n)}(y, z, St)$  will commence soon.



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